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CORRECTIONS OF ELEMENTS OF BOMBING TABLES FOR EFFECTS OF HEIGHT OF TARGET

BY

E. J. MCSHANE

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Abstract

Two possible procedures for computing the effect of height of target on range or trail are discussed. The simpler method is shown to yield corrections differing very little from those obtained by the theoretically sounder but more complicated method. It is, therefore, recommended for use.

INTRODUCTION

1. In conversations with Dr. H. E. Heinecke, and in correspondence O.O. 063.2/1541 APG 353.42/1205, it has been brought out that many air fields at which bombardiers are trained have altitudes between 3000 and 6000 feet above sea level. Since bombing tables are prepared for targets at sea level, a systematic error is introduced by the use of such tables at the fields having high altitudes. It was, therefore, suggested that the Ballistic Research Laboratory prepare tables permitting correction for height of target above sea level.

The present report contains a discussion of the theoretical basis for the computation of such correction tables. The computations have already been completed, under the direction of Mr. E. S. Martin. The principal conclusion is that there is no point in adopting any procedure of greater theoretical refinement than the pure-drag solution presented in section I.

I. The Pure-Drag Solution

If the air produced no effect on the motion of the bomb except through drag, and the drag coefficient were accurately known, it would be very easy to produce such tables.

Suppose that a bomb is launched from horizontal flight at an altitude Y feet above sea level and speed v_o feet per second. Suppose also that the drag function is $G(v)$ and the reciprocal ballistic coefficient is γ, and the air z feet above sea level has the ballistic standard density

$$\rho = \rho_o e^{-az}$$

$$(\rho_o = .0751 \text{ lb/ft.}^3, \quad a = .0003158 \text{ ft.}^{-1}).$$

Choose axes with origin at the point of release, x-axis horizontal in the direction of flight, and y-axis vertically downward. The trajectory is then the solution of the equations:

$$(1.1) \qquad \ddot{x} = -\gamma e^{-a(Y-y)} G(v)\dot{x},$$

$$\ddot{y} = g - \gamma e^{-a(Y-y)} G(v)\dot{y}$$

with initial conditions

$$(1.2) \qquad x(0) = y(0) = \dot{y}(0) = 0, \quad \dot{x}(0) = v_o .$$

(As usual, the factor ρ_o is incorporated in G, and v denotes the air speed of the bomb.)

Now consider a bomb aimed at a target Y_ω feet above sea level, launched from an airplane in horizontal flight Y feet above the target (and, therefore, $Y + Y_\omega$ feet above sea level) at speed v_o feet per second. Axes are chosen as before, with origin at point of release, x-axis in line of flight and y-axis vertically down. The point with coordinates (x,y) is now at altitude $Y + Y_\omega - y$ feet above sea level, so air density at (x,y) is $e^{-a(Y + Y_\omega - y)}$ times standard density at sea level. The differential equations of the trajectory are:

$$(1.3) \qquad \ddot{x} = -\gamma e^{-a(Y + Y_\omega - y)} G(v)\, \dot{x} ,$$

$$\ddot{y} = g - \gamma e^{-a(Y + Y_\omega - y)} G(v)\dot{y} ,$$

with initial conditions (1.2). But equations (1.3) can be written in the form

$$(1.4) \qquad \ddot{x} = -(\gamma e^{-aY_\omega})\, e^{-a(Y-y)} G(v)\, \dot{x} ,$$

$$\ddot{y} = g - (\gamma e^{-aY_\omega})\, e^{-a(Y-y)} G(v)\, \dot{y} .$$

These are equations (1.1) with only the change that the constant γ is replaced by the constant

$$\gamma e^{-aY_\omega} .$$

Hence, if the drag function $G(v)$ is accurately known and the air produces no effect except drag on the bomb, the trajectory of a bomb with reciprocal ballistic coefficient γ launched Y feet above a target Y_ω feet above sea level will be exactly like the trajectory of a bomb with reciprocal ballistic coefficient

$$\gamma e^{-aY_\omega}$$

launched Y feet above sea level, and with the same speed.

II. CORRECTIONS BASED ON PURE-DRAG SOLUTION

The effect of replacing γ by

$$\gamma e^{-aY_\omega}$$

is to increase the range of the bomb and decrease its time of fall. However, it would be quite inconvenient for a bombardier to have to interpolate for two different small corrections, one to trail and one to time of fall or DS, and set both corrections on the bomb sight. In the absence of cross-wind, a correction to trail alone can be given which will produce the correct point of impact. Let RP_1 be the track of the airplane; in

R M P_1P_2

the absence of cross-wind, this passes over the target M to obtain a hit. Let P_1 be the point vertically below the airplane at the instant of impact. For a hit, MP_1 must equal the trail λ_1 of the bomb. Let R be the point vertically below the airplane at the instant of release. Then RP_1 is the travel of the airplane during the time of flight T_1 of the bomb. If v_g is the ground speed of the airplane,

$$RP_1 = v_gT_1, \quad \text{so}$$

$$RM = v_gT_1 - \lambda_1 .$$

If an incorrect time of flight T_2 is used instead of T_1, impact will occur when the airplane is vertically above P_2, where

$$RP_2 = v_gT_2 \quad . \quad \text{Hence}$$

$$P_1P_2 = R(T_2 - T_1) .$$

A hit will still be obtained if $\lambda_2 = MP_2$ is set in as the trail instead of the correct value MP_1. For then

$$RM = v_gT_2 - \lambda_2 .$$

To express it more simply, in still air it is immaterial whether the correct trail and time of flight are used, provided only that

$v_gT - \lambda$ is equal to the true value of the range X = RM of the bomb. The effect of raising the target above sea level is to increase the range by an amount ΔX. In still air, a hit will be obtained if $v_gT - \lambda$ is also increased by an amount ΔX. This can be done correctly, but inconveniently, by reducing both T and λ to their new value. More conveniently, it can be done by decreasing λ by the amount ΔX. This simplified process leads to no error in still air.

It remains to investigate the error caused by this simplification when there is a wind. Let W̲ be the vector representing the wind at release. The situation, like many others involving winds, can be rather easily visualized by choosing axes fixed with respect to the air at release. In these axes there is no wind, but the ground is sliding by the axis-system with velocity -W̲. The situation is thus mathematically the same as in bombing a uniformly moving target in still air. The Norden bomb sight takes correct account of such target motion or wind (apart from an error called the "range component of cross trail" which is insignificant except for light bombs in high cross-winds), provided that the correct time of flight is used. The signal for release is given when the relative position of airplane and target is such that T seconds later (T = time of flight of bomb) the target will have reached the point of impact. If, however, the time of flight is falsely assumed to be $T + \Delta T$, release will take place at such an instant that $T + \Delta T$ seconds later the target will have reached the point of impact. But the bomb reaches this point T seconds after release, at which time the target has a vector displacement $(-\underline{W})(-\Delta T) = \underline{W}\,\Delta T$ from the point of impact. The effect of height of target is to produce a negative ΔT. So the bomb will strike upwind of the target, at a distance equal to the distance the release wind travels in time $|\Delta T|$.

The magnitude of this error may be seen from the following brief table in which Y is the altitude above target in feet, h_t is 5000 feet and ΔT is the change in time of flight of a 100-lb. Bomb, Practice, M38A2.

ΔT:

Y \ v, mi/hr	150	400
10,000	-.15	-.20
20,000	-.37	-.42
30,000	-.57	-.63

From this it follows that the target error, in feet, due to a 50 mi/hr wind at release is as follows:

Y \ v	150	400
10,000	11	15
20,000	27	31
30,000	42	46

or in mils is

Y \ v	150	400
10,000	1.1	1.5
20,000	1.4	1.6
30,000	1.4	1.5

It would not seem practical to reduce this small error by further complicating the bombardier's task.

III. ERRORS INHERENT IN THE PURE-DRAG SOLUTION

The pure-drag solution in section I is based on two false assumptions, namely that the drag is accurately known and that no aerodynamic forces other than the drag produce appreciable effects. The former can be further subdivided into the two errors that the drag function is know for standard temperature and that the temperature structure is the same between Y_ω and $Y + Y_\omega$ as between 0 and Y. This last may be disposed of at once. This last may be disposed of at once. The effect of temperature may indeed be appreciable, but it would be absurd to consider it in a small correction term while its effect on the data in the main table is ignored. At some future date, temperature corrections to trail and time of flight may be made. But then these will be corrections to the tabulated trail and time of flight, and it is hardly to be anticipated that the small correction for height of target will ever need to be itself corrected for temperature.

A somewhat similar statement applies to the error in the drag function. At present all bombing tables are obtained with the help of ballistic tables, based on the Gâvre drag function, and there are many indications that this is not even a fair approximation to the true drag function for any bomb in present use. Consequently, all corrections based on the Gâvre drag function are subject to suspicion. But this is far less important than the fact that uncertainty also prevails in the bombing table itself. When a better drag function is determined, the outstanding use of it will be to improve extrapolation in bombing tables, and only after that to improve correction terms. In any case, it is out of the question to try to correct for erroneous drag function in the height-of-target correction, since as yet, no adequately trustworthy drag function has been determined.

The other false assumption in section I is that no aerodynamic force except lift is worth considering. The lift force also produces very perceptible effects. This will now be discussed.

IV. EFFECT OF STEADY LIFE ON HEIGHT-OF-TARGET CORRECTION

It has been known for some years, as a result of range bombings at the Aberdeen Proving Ground, that the drag is not the only aerodynamic force which appreciably affects the motion of the bomb. There is also a force perpendicular to the direction of motion of the bomb. This has been investigated in some detail in Ballistic Research Laboratory Report No. 325, "The Plane Yawing of Bombs Launched from Horizontal Suspension". The effect is produced almost entirely by a slowly varying component of force in the vertical plane containing the trajectory and perpendicular to the

trajectory, and with magnitude

$$M \lambda (\rho/\rho_0) \cos\theta \ ,$$

where θ is the angle between the horizontal and the tangent to the trajectory, λ is a constant depending on the bomb and with the dimensions of an acceleration and M is the mass of the bomb.

The coefficient λ can be evaluated from the results of range bombing at the 2000 foot level.* The result at present has not as great accuracy as might be desired, but it is hoped that improved equipment will help in this respect. Never-the-less, the values given in the report cited are at least of the right order of magnitude. It appears that the value of λ for the M38A2 practice bomb is greater than for any other, and is about .011g.

The equations of motion, when the steady lift is included, take the form

$$(4.1) \quad \ddot{x} = -\gamma e^{-a(Y-y)} G(v)\,\dot{x} + \lambda e^{-a(Y-y)} \dot{x}\dot{y}/u^2 \ .$$

$$\ddot{y} = g - \gamma e^{-a(Y-y)} G(v)\,\dot{y} - \lambda e^{-a(Y-y)} \dot{x}^2/y^2 .$$

As before, consider a bomb launched from a height $Y + Y_\omega$ above a target whose altitude is Y_ω. The differential equations of the trajectory become

$$(4.2) \quad \ddot{x} = -\gamma e^{-a(Y_\omega + Y-y)} G(v)\dot{x} + \lambda e^{-a(Y_\omega + Y-y)} \dot{x}\dot{y}/u^2 \ ,$$

$$\ddot{y} = g - \gamma e^{-a(Y_\omega + Y-y)} G(v)\dot{y} - \lambda e^{-a(Y_\omega + Y-y)} \dot{x}^2/u^2 .$$

These can be written in the form

$$(4.3) \quad \ddot{x} = -(\gamma e^{-aY_\omega})\, e^{-a(Y-y)} G(v)\dot{x} + (\lambda e^{-aY_\omega}) e^{-a(Y-y)} \dot{x}\dot{y}/u^2 \ ,$$

$$\ddot{y} = g - (\gamma e^{-aY_\omega}) e^{-a(Y-y)} G(v)\dot{y} - (\lambda e^{-aY_\omega}) e^{-a(Y-y)} \dot{x}^2/u^2 \ ,$$

These are equations (4.1) with the change that γ, λ are replaced by

* Ballistic Research Laboratory Report No. 337, "Separation of Lift and Drag Effects on Bomb Trajectories by Analysis of Range Bombing Data."

$$\gamma e^{-aY_\omega}, \quad \lambda e^{-aY_\omega} \qquad \text{respectively.}$$

The effects of the steady lift are too large to ignore, but are small enough to be accurately treated as differential corrections to the pure-drag trajectory. A table of the effect of lift on range and on time of flight has been computed by the method set forth in Ballistic Research Laboratory Report No. 327, and will be found at the end of this present report. This table is not extensive enough for all purposes, and since it will later be extended, it has not been put in final form. However, it is adequate for the present discussion. The argument $\log C_s$ is the logarithm to base 10 of the "summital ballistic coefficient", which is $1/\gamma e^{-aY}$ when the launching is at Y feet above sea level. Hence

$$(4.4) \qquad \log C_s = aY \log_{10} e - \log_{10}\gamma$$

$$= .000013716Y - \log_{10}\gamma,$$

if Y is in feet.

Now, if the drag function were accurately known, a correct procedure for determining the effect of a height of target Y_ω would be the following. First, using the reciprocal ballistic coefficient γ_x determined by the range, find $\log C_s$ by (4.4), and find the effect of lift $\lambda = .01g$ on range. Multiply this by $\lambda/.01g$ (which for the 100-lb. M38A2 practice bomb is 1.1) and subtract from the observed range. This gives the range corrected for lift. From the Ballistic Reduction Tables find the corresponding γ. Properly, this should be repeated using this new γ in place of γ_x, but in fact, the change in lift effect due to the correction to γ is negligibly small. Next multiply the corrected γ by

$$e^{-aY_\omega},$$

and from the Ballistic Reduction Tables find the range corresponding to this new γ. From the table of lift effects, find the effect on range corresponding to reciprocal ballistic coefficient

$$\gamma e^{-aY_\omega}$$

or

$$(4.5) \qquad \log C_s = .000013716(Y + Y_\omega) - \log_{10}\gamma.$$

Multiply this by

$$(\lambda e^{-aY_\omega})/.01g,$$

and add to the range found from the Ballistic Reduction Tables. The difference between this sum and the range for target at sea level is the range

correction sought, and as seen in section II can be used as the correction to trail.

If the drag function were accurately known, the process of correcting γ_t for lift effect described in the beginning of the preceding paragraph would lead (except for experimental error) to the same value of γ as would the exactly analogous process of correcting γ_x for lift effect. Since the Gâvre drag function is not correct for bombs, there will still be a discrepancy between the ballistic coefficients for time and range, even after correcting for lift. As previously stated, nothing can be done about this at present, and so the ballistic coefficient for time will be ignored.

The process just described, although accurate, is rather cumbersome to use. Its form will, therefore, be modified. To begin with, it is convenient to replace the range X by the range lag

$$B = \text{vacuum range} - X.$$

Since the velocity at launching and the height above target are unchanged in making the correction, the vacuum range is unaltered by elevating the target. So the increase in range is equal to the decrease in range lag. Also, since speed and altitude are fixed, B depends only on γ. The B corresponding to a given γ will be denoted by $B(\gamma)$, and the γ corresponding to a given B will be denoted by $\gamma(B)$. Also, $B'(\gamma)$ will mean $dB/d\gamma$ and $\gamma'(B)$ will mean $d\gamma/dB$, and $B''(\gamma)$ will mean $d^2B/d\gamma^2$.

Let B_o be the range lag for the bomb launched at the given speed at a height Y feet above a target which is at sea level, and let l_o be the additional range due to lift effect. Then the range lag which drag alone would produce would be $B_o + l_o$, and the reciprocal ballistic coefficient (for range) as corrected for lift is

$$\gamma(B_o + l_o),$$

$$\alpha = e^{-Y\omega}, \tag{4.6}$$

Consider now the trajectory of a bomb dropped from Y feet above a target at level $Y\omega$. As has been shown, the range lag due to drag is that which corresponds to reciprocal ballistic coefficient $\alpha\gamma(B_o + l_o)$, that is $B(\alpha\gamma(B_o + l_o))$. The new lift effect is found by computing the lift effect $l(\alpha\gamma)$ corresponding to the new reciprocal ballistic coefficient $\alpha\gamma(B_o + l_o)$ and multiplying by α; it will be denoted by $\alpha l(\alpha\gamma)$. The range lag is then

$$B(\alpha\gamma(B_o + l_o)) - \alpha l(\alpha\gamma).$$

The pure-drag method expounded in section I would lead to the new range lag

$$B(\alpha\gamma(B_o)).$$

Hence, the error in the pure-drag method due to ignoring the lift effect

is

$$(4.7) \qquad \varepsilon = B(\alpha\gamma(B_o)) - B(\alpha\gamma(B_o + l_o)) + \alpha\, l(\alpha\gamma)$$

$$= B(\alpha\gamma(B_o)) - B(\alpha\gamma(B_o + l_o)) + \alpha\, l_o + \alpha\left[l(\alpha\gamma) - l_o\right].$$

The first two terms represent the difference of the values of the function $B(\alpha\gamma(B_o + z))$ at $z = 0$ and at $z = l_o$. By the theorem of mean value, this is $-l_o$ times the derivative of the function at some point $\bar{z}$ between 0 and l_o. But

$$\frac{d}{dz} B(\alpha\gamma(B_o + z)) = B'(\alpha\gamma(B_o z)) \,.\alpha\gamma'(B_o + z).$$

Hence

$$(4.8) \quad \varepsilon = \alpha l_o \left\{1 - B'(\alpha\gamma(B_o + \bar{z}))\,\gamma'(B_o + \bar{z})\right\} + \alpha\left[l(\alpha\gamma) - l_o\right].$$

By the theorem of mean value,

$$B'(\alpha\gamma(B_o + \bar{z})) = B'(\gamma(B_o + \bar{z})) + (\alpha - 1)\,\gamma(B_o + \bar{z})B''(\bar{\alpha}\gamma(B_o + \bar{z}))$$

where $\bar{\alpha}$ is between α and 1. On substituting this in (4.8) and recalling that

$$\gamma'(B_o + \bar{z}) = 1/B'(\gamma(B_o + \bar{z})) ,$$

it is found that

$$(4.9) \qquad \varepsilon = \alpha(1 - \alpha)l_o\, \gamma(B_o + \bar{z})\, B''(\bar{\alpha}\gamma(B_o + \bar{z}))/B'(\gamma(B_o + \bar{z}))$$

$$+ \alpha\left[l(\alpha\gamma) - l_o\right] .$$

Inspection of the tables shows that B' is positive and B'' negative, while $l(\alpha\gamma)$ is greater than l_o. Hence, the two terms in (4.9) have opposite signs, and the error is numerically less than the numerically larger of the two terms on the right. In order to discuss a fairly extreme case, it will be supposed that $Y\omega$ = 5000 feet, whence

$$\alpha = .8539 \quad .$$

Also, elevation by 5,000 feet increases C_s by .06818. Inspection of the table of lift effects shows that for lift coefficient .01g a change of log C_s by .06818 does not affect the range correction by more than about 2 feet. Hence, for λ = .011g (which is the value corresponding to the 100-lb. M38A2 practice bomb, and is apparently not exceeded for any G.P. or A.P. bomb) the term $\alpha\left[l(\alpha\gamma) - l_o\right]$ remains under 2 feet. If it is numerically the greater of the two terms on the right of (4.9), ε is negligible; otherwise, the second term merely serves to reduce the effect of the first term.

Both B' and B'' decrease numerically as γ increases, so

$$(4.10) \qquad \left| B''(\alpha\gamma(B_o + \bar{z}))/B'(\gamma(B_o + \bar{z})) \right| \doteq \left| B''(\alpha\gamma(B_o + \bar{z}))/B'(\gamma(B_o + \bar{z})) \right| .$$

Both l_o and the right member of (4.10) increase with speed of launching, so they will be estimated at 480 miles per hour. The argument γ in the Ballistic Reduction Tables is tabulated at intervals of .1, and it is quite adequate to consider that $B'(\gamma)$ is ten times the mean first difference, and $B''(\gamma)$ is one hundred times the central second difference. Computation shows that $-\gamma B''/B'$ does not exceed .462 for $\gamma \leqq 1.3$, and does not exceed .528 for $\gamma \leqq 1.9$. For $\lambda/.01g = 1.1$ the lift effect on range does not exceed 105 feet for any entry in the table. So for $\gamma \leqq 1.3$ the value of the first term in (4.9) can never exceed 6.05 feet. The same estimate will continue valid past $\gamma = 1.9$ because of the decrease of lift effect on range with increasing γ. Moreover, this 6.05 feet is still to be diminished by the small term

$$\alpha\left[l(\alpha\gamma) - l_o\right].$$

At lower altitudes and speeds, the error is still smaller. For example, at altitude 20,000 feet above target and speed 320 mi/hr. true, it is less than 2.5 feet for all bombs having $\gamma \leqq 1.0$.

It is, therefore, clear that it would be pointless to complicate the computation of the table of corrections for height of target by considering lift effect separately from drag effect. Accordingly, the tables of height-of-target corrections already computed for the 100-lb. M38A2 practice bomb were prepared by the pure-drag method of section 1, and this method will also be quite accurate enough for use in preparing height-of-target correction tables for any bomb.

E. J. Mc Shane

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EFFECTS OF STEADY LIFT WITH $\lambda = .01g$

TRUE AIR SPEED 160 mi/hr

Y \ Log C_s	Range Effect, Feet 0.0	0.4	0.8	1.2	Time of Flight Effect, seconds 0.0	0.4	0.8	1.2
2000	15	16	16	16	.042	.041	.041	.041
4000	23	24	25	25	.049	.048	.048	.048
6000	29	30	31	31	.052	.051	.051	.050
8000	33	35	36	37	.053	.052	.051	.050
10000	36	39	40	40	.052	.052	.050	.050
12000	39	42	44	44	.051	.050	.049	.049
14000	41	45	47	47	.049	.048	.048	.048
16000	43	37	38	39	.048	.047	.046	.047
18000	44	48	50	51	.045	.045	.044	.045
20000	44	49	51	52	.044	.043	.043	.043
22000	44	49	52	53	.041	.041	.041	.041
24000	44	50	52	53	.039	.039	.039	.039
26000	43	50	52	54	.037	.038	.037	.037
28000	43	50	52	54	.035	.036	.036	.035
30000	41	49	52	54	.033	.034	.034	.034
32000	40	48	52	53	.032	.032	.032	.032
34000	39	47	51	53	.030	.030	.030	.030
36000	38	47	50	52	.028	.029	.029	.029
38000	36	46	50	51	.027	.027	.028	.028
40000	35	44	49	50	.025	.026	.026	.026
42000	33	43	48	49	.024	.024	.025	.025
44000		42	46	48		.023	.024	.023
46000		41	45	47		.022	.023	.022
48000		40	44	46		.021	.022	.021
50000		38	43	45		.020	.021	.020
52000		37	42	44		.019	.020	.019
54000		35	40	43		.017	.019	.018
56000			39	41			.017	.017
58000			38	40			.016	.016
60000			37	39			.016	.015
62000			35	38			.015	.015
64000			34	37			.014	.014
66000				35				.013
68000				34				.013
70000				33				.012
72000				32				.012

EFFECTS OF STEADY LIFT WITH $\lambda = .01g$

TRUE AIR SPEED 320 mi/hr

Y \ Log C_s	Range Effect, Feet 0.0	0.4	0.8	1.2	Time of Flight Effect, seconds 0.0	0.4	0.8	1.2
2000	23	26	26	27	.051	.050	.050	.049
4000	33	36	38	38	.063	.063	.062	.061
6000	40	44	46	47	.071	.069	.069	.067
8000	45	50	53	54	.075	.073	.071	.070
10000	49	55	58	59	.077	.074	.072	.071
12000	52	59	62	64	.076	.074	.073	.071
14000	55	62	66	67	.075	.073	.071	.071
16000	56	65	68	70	.074	.072	.070	.069
18000	57	66	70	72	.071	.070	.069	.067
20000	60	68	72	74	.069	.068	.067	.066
22000	58	68	73	75	.066	.066	.065	.064
24000	58	69	74	76	.063	.064	.062	.061
26000	58	69	74	77	.060	.062	.060	.059
28000	57	69	74	76	.057	.059	.058	.057
30000	55	68	74	76	.054	.057	.055	.055
32000	54	68	74	76	.051	.054	.054	.052
34000	53	67	73	75	.047	.052	.051	.051
36000	51	65	72	74	.045	.050	.049	.048
38000	49	64	71	73	.042	.048	.047	.046
40000	47	63	70	73	.040	.045	.044	.044
42000	46	61	68	71	.038	.043	.042	.042
44000		60	67	70		.041	.040	.040
46000		58	65	69		.038	.038	.039
48000		57	64	67		.036	.036	.037
50000		54	63	66		.034	.034	.035
52000		53	61	64		.032	.033	.034
54000		51	60	63		.031	.031	.032
56000			58	61			.030	.031
58000			56	60			.028	.029
60000			55	58			.027	.028
62000			54	57			.026	.026
64000			51	55			.025	.025
66000				53				.024
68000				52				.022
70000				50				.021
72000				48				.020

EFFECTS OF STEADY LIFT WITH $\lambda = .01g$

TRUE AIR SPEED 480 mi/hr

Y \ Log C_g	0.0	0.4	0.8	1.2	0.0	0.4	0.8	1.2
2000	32	36	38	38	.055	.053	.052	.052
4000	42	48	52	53	.072	.069	.067	.066
6000	49	58	61	63	.082	.079	.076	.074
8000	54	64	69	71	.087	.083	.081	.079
10000	58	69	75	77	.090	.086	.084	.082
12000	61	73	79	83	.090	.087	.085	.084
14000	63	76	84	87	.090	.087	.085	.084
16000	65	78	86	89	.089	.087	.084	.083
18000	66	80	88	92	.087	.086	.083	.082
20000	66	81	89	94	.084	.084	.082	.080
22000	66	82	90	94	.081	.082	.080	.078
24000	66	82	91	95	.078	.079	.078	.076
26000	66	82	92	95	.075	.077	.076	.074
28000	65	82	92	95	.072	.074	.075	.072
30000	64	81	91	95	.068	.071	.073	.070
32000	62	80	90	94	.065	.069	.070	.068
34000	60	79	89	93	.062	.066	.067	.065
36000	59	78	88	92	.059	.063	.064	.063
38000	57	77	87	91	.057	.061	.061	.061
40000	55	75	85	90	.054	.058	.059	.059
42000	53	74	84	88	.051	.055	.056	.056
44000		72	82	87		.053	.054	.054
46000		70	80	85		.051	.052	.052
48000		68	79	83		.049	.050	.050
50000		66	77	82		.047	.048	.048
52000		64	75	80		.044	.047	.046
54000		62	73	78		.042	.045	.044
56000			71	76			.043	.042
58000			69	75			.042	.041
60000			67	73			.041	.038
62000			66	71			.039	.037
64000				69				.035
66000				68				.033
68000				65				.032
70000				60				.028